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The response of self-graviting protostellar discs to slow reduction in cooling timescale: the fragmentation boundary revisited

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ABSTRACT

A number of previous studies of the fragmentation of self-gravitating protostellar discs have involved suites of simulations in which radiative cooling is modeled in terms of a cooling timescale ($t_{\rm cool}$) which is parameterised as a simple multiple ($\beta_{\rm cool}$) of the local dynamical timescale. Such studies have delineated the 'fragmentation boundary' in terms of a critical value of $\beta_{\rm cool}$ ($\beta_{\rm crit}$) such that the disc fragments if $\beta_{\rm cool} < \beta_{\rm crit}$. Such an approach however begs the question of how in reality a disc could ever be assembled in a state with $\beta_{\rm cool} < \beta_{\rm crit}$. Here we adopt the more realistic approach of effecting a gradual reduction in $\beta_{\rm cool}$, as might correspond to changes in thermal regime due to secular changes in the disc density profile. We find that the effect of *gradually* reducing $\beta_{\rm cool}$ (on a timescale longer than $t_{\rm cool}$) is to stabilise the disc against fragmentation, compared with models in which $\beta_{\rm cool}$ is reduced rapidly (over less than $t_{\rm cool}$). We therefore conclude that the ability of a disc to remain in a self-regulated, self-gravitating state (without fragmentation) is partly dependent on the disc's thermal history, as well as its current cooling rate. Nevertheless, the effect of a slow reduction in $t_{\rm cool}$ appears only to lower the fragmentation boundary by about a factor two in $t_{\rm cool}$ and thus only permits maximum 'α' values (which parameterise the efficiency of angular momentum transfer in the disc) that are about a factor two higher than determined hitherto. Our results therefore do not undermine the notion that there is a fundamental upper limit to the heating rate that can be delivered by gravitational instabilities before the disc is subject to fragmentation. An important implication of this work, therefore, is that self-gravitating discs can enter into the regime of fragmentation via *secular* evolution and it is not necessary to invoke rapid (impulsive) events to trigger fragmentation.

Key words: accretion, accretion discs – star: formation – gravitation – instabilities – stars: formation

1 INTRODUCTION

Following the seminal work of Gammie (2001), there has been considerable progress in recent years in understanding the behaviour of self-gravitating accretion dics (see Durisen et al. 2007 and references therein). A number of simulations (Gammie 2001; Rice et al. 2003; Lodato & Rice 2004, 2005) have demonstrated that if the thermodynamic properties of the disc are evolved according to a thermal equation (involving a cooling term parameterised in terms of a cooling timescale, $t_{\rm cool}$), then the disc may be able to establish a *self-gravitating, self-regulated* state. In this state, the Toomre Q parameter:

$$Q = \frac{c_{\rm s}\kappa}{\pi G \Sigma}, \quad (1)$$

(where $c_{\rm s}$ is the sound speed, κ is the epicyclic frequency (equal to the angular velocity Ω in a Keplerian disc) and Σ is the disc surface density) hovers at a value somewhat greater than unity over an extended region of the disc. Whereas the state $Q = 1$, corresponds to a situation of marginal stability against *axisymmetric* perturbations, in the self-regulated state the disc is instead subject to a variety of non-axisymmetric self-gravitating modes whose effect, through the action of weak shocks, is to dissipate mechanical energy (i.e. kinetic and potential energy of the accretion flow) as heat. Thermal equilibrium is then attained through the balancing of such heating by the prescribed radiative cooling: in essence, self-regulation results when the amplitude of these modes is able to self-adjust so as to maintain thermodynamic equilibrium against the relevant energy loss processes.

The above studies have all found, however, that such self-regulation is only possible in the case that the cooling timescale is not too short: stability demands that $\beta_{\rm cool} = t_{\rm cool}/\Omega^{-1}$ exceeds a critical value which, for discs with adiabatic index of 5/3, is ~ 7 (Rice et al. 2005). In the case of more rapid cooling, the disc instead fragments.

Such simulations however approach the 'fragmentation boundary' in a manner that is unlikely ever to apply to discs in reality. In the simulations, the discs are set up without additional heating mechanisms and are subject to cooling at some prescribed value of $\beta_{\rm cool}$. Discs with $\beta_{\rm cool} < \beta_{\rm crit}$ then fragment on the local cooling timescale (i.e. a few times the local dynamical timescale), thus begging the question of how such unstable initial conditions could ever have been set up in the first place.

A more likely scenario for disc fragmentation is that the disc is instead set up in self-regulated, self-gravitating state and then conditions *gradually* change so that $\beta_{\rm cool}$ is lowered. (For example, continued infall of material onto a disc or secular re-arrangement of material in the disc due to the action of gravitational torques could alter the surface density profile of the disc and allow it to enter a new cooling regime with lower $\beta_{\rm cool}$). It is not however clear that the fragmentation boundary would be the same in the case that $\beta_{\rm cool}$ is gradually reduced.

In this paper we conduct a suite of idealised simulations in which we explore whether the fragmentation boundary just depends on the instantaneous value of $\beta_{\rm cool}$ (as has been assumed hitherto) of whether the system 'remembers' the history of how it evolved to a point of given $\beta_{\rm cool}$. Such a ('toy model') approach, is complementary to studies (Boley et al. 2006; Mayer et al. 2007; Stamatellos et al. 2007, see also the analytical estimates by Rafikov 2005; Rafikov 2007) which attempt to achieve ever-increasing verisimilitude via the incorporation of more realistic treatments of radiative transfer. Here, instead, we make no claims that the simplified cooling law (for example, the assumption that $\beta_{\rm cool}$ is spatially uniform) actually corresponds to a situation encountered in a real disc, because our aim is to isolate a particular physical effect (i.e. the timescale on which the fragmentation boundary is approached). The computational expense of 'realistic' simulations however prevents their use to study secular effects: even in the case of the present 'toy' simulations, it is impracticable to run simulations over the long timescales on which the Σ profile changes due to gravitational torques or infall. We can nevertheless assess the effect of relatively slow changes in $\beta_{\rm cool}$ on the fragmentation boundary through imposing an *ad hoc* reduction in the value of $\beta_{\rm cool}$ and can apply this insight to the secular evolution of real discs.

In particular, we want to examine the cause of the fragmentation for $\beta_{\rm cool} < \beta_{\rm crit}$, that has been found in previous simulations. Is this (i) due to the disc's inability to maintain - under any circumstances - a gravitational heating rate that can match the imposed high cooling rate? This is the hypothesis of Lodato & Rice (2005), who identify the minimum value of $\beta_{\rm cool}$ with a maximum value of the gravitationally induced angular momentum transfer that can be delivered by a disc without its fragmenting. They parameterise this state of maximal angular momentum transfer in terms of the ratio of the r, ϕ component of the stress tensor to the thermal pressure, i.e., by analogy with the equivalent expression for a *viscous* disc, in terms of a maximum in the well known viscous 'α' parameter (Shakura & Sunyaev 1973). A critical value of $\beta_{\rm cool}$ of ~ 7 corresponds to a maximum α of ~ 0.06.

Alternatively, (ii) does fragmentation instead reflect the disc's inability to set up the required high heating rate *on the short timescale* ($t_{\rm cool}$) on which the disc is cooling? If this were the case, then with sufficiently gradual approach to the regime of low $\beta_{\rm cool}$, the disc could in principle deliver a value of α that exceeded the above limit by a generous margin.

We can obviously distinguish between these alternatives by investigating the case in which $\beta_{\rm cool}$ is reduced on a timescale τ that is longer than $t_{\rm cool}$, since in this case the disc temperature will fall via a sequence of thermal equilibrium states (on timescale τ), rather than dropping on timescale $t_{\rm cool}$. The aim of this investigation is thus to see whether the disc is more resistant to fragmentation in the regime that $\tau > t_{\rm cool}$. If it is *not*, then the manner in which the disc approaches the fragmentation boundary is unimportant. If, on the other hand, it is found that rapid changes in cooling regime are required, then it may be necessary to invoke impulsive events (such as an external dynamical interaction) to trigger fragmentation.

In Section 2 we describe the numerical setup, discuss our results in Section 3 and in Section 4 we present some conclusions.

2 NUMERICAL SETUP

2.1 The SPH code

Our three-dimensional numerical simulations are carried out using SPH, a Lagrangian hydrodynamic scheme (Benz 1990; Monaghan 1992). The general implementation is very similar to Lodato & Rice (2004), Lodato & Rice (2005) and Rice et al. (2005). The gas disc is modeled with 250,000 SPH particles (500,000 in a run used as a convergence test) and the local fluid properties are computed by suitably averaging over the neighbouring particles. The disc is set in almost Keplerian rotation (allowing from slight departures from it to account for the effect of pressure forces and of the disc gravitational force) around a central point mass onto which gas particles can accrete if they get closer than the accretion radius, taken to be equal to 0.5 code units.

The gas disc can heat up due to $p\mathrm{d}V$ work and artificial viscosity. The ratio of specific heats is $\gamma = 5/3$. Cooling is here implemented in a simplified way, i.e. by parameterizing the cooling rate in terms of a cooling timescale:

$$\left(\frac{\mathrm{d}u_{\rm i}}{\mathrm{d}t}\right)_{\rm cool} = -\frac{u_{\rm i}}{t_{\rm cool}}, \tag{2}$$

where $u_{\rm i}$ is the internal energy of a particle and the cooling timescale $t_{\rm cool}$ is assumed to be proportional to the dynamical timescale, $t_{\rm cool} = \beta_{\rm cool}\Omega^{-1}$, where $\beta_{\rm cool}$ is varied according to a time-dependent prescription (see Section 2.3 below).

Artificial viscosity is introduced using the standard SPH formalism. The actual implementiation is very similar to the one used in Rice et al. (2005), that is we set the two relevant numerical parameters to $\alpha_{\rm SPH} = 0.1$ and $\beta_{\rm SPH} = 0.2$ and we have not included here (consistent with Rice et al. 2005) the so-called Balsara switch (Balsara 1995) to reduce shear viscosity.

2.2 Disc setup

The main physical properties of the disc at the beginning of the simulation are again similar to those of Lodato & Rice (2004, 2005). The disc surface density Σ is initially proportional to R^{-1} (where R is the cylindrical radius), while the temperature is initially proportional to $R^{-1/2}$. Given our simplified form of the cooling function, the computations described here are essentially scale free and can be rescaled to different disc sizes and masses. For reference, we will assume that the unit mass (which is the mass of the central star) is

Simulation	x	$\beta_{\rm hold}$	N	fragmentation
F1	10.5	—	250K	yes
F2	10.5	3	250K	yes
V	105-10.5	3	250K	yes
S1	105	3	250K	no
Sh	105	3	500K	yes
S2	105	2.75	250K	no
S3	105	2.62	250K	yes
VS1	314	3	250K	no
VS2	314	2.75	250K	yes

Table 1. Details of the various simulations discussed in this paper. The different columns indicate: the name of the run, the value of the parameter x determining the speed of the reduction of the cooling time, the value of $\beta_{\rm hold}$ (if any) at which the cooling time was held fixed after reduction, the number of particles used in the run N and whether fragmentation did occur or not. Simulation V was performed with an initially slow reduction of β (with $x = 105$), followed by a fast reduction (with $x = 10.5$), so that it would reach $\beta = 3$ with a fast reduction at the same time as simulation S1.

$1M_{\odot}$ and that the unit radius is $1AU$. In this units the disc extends from $R_{\rm in} = 0.25AU$ to $R_{\rm out} = 25AU$. The normalization of the surface density is generally chosen such as to have a total disc mass of $M_{\rm disc} = 0.1M_{\odot}$, while the temperature normalization is chosen so as to have a minimum value of $Q = 2$, which is attained at the outer edge of the disc.

Initially, the disc is evolved with constant $\beta_{\rm cool} = 7.5$, this value of $\beta_{\rm cool}$ being in the regime where previous work Gammie (2001), Rice et al. (2005) has shown that the disc does not fragment. The general features of this initial evolution is described in detail in Lodato & Rice (2004). The disc starts cooling down until the vertical scale-length H is reduced such that $H/R \approx M_{\rm disc}/M_{\star} = 0.1$. At this point the disc becomes Toomre unstable and develops a spiral structure that heats up the disc and maintains it close to marginal stability. We have evolved the disc with this value of $\beta_{\rm cool}$ for 7.8 outer disc orbits. At this stage it is close to $Q = 1$ over most of the disc (i.e. over the radial range $R = 3-23$ A.U.).

2.3 Evolution of $\beta_{\rm cool}$

After evolution of the disc with $\beta_{\rm cool} = \beta_{\rm cool}(0) = 7.5$ for a cooling timescale, we effect a linear reduction of $\beta_{\rm cool}$ on a timescale T, i.e.

$$\beta_{\rm cool}(t) = \beta_{\rm cool}(0)\left(1 - \frac{t}{T}\right) \quad (3)$$

where we set $T = x\Omega^{-1}(R_{out})$.

Such a prescription implies that the timescale $\tau(t)$ on which the local instantaneous value of $t_{\rm cool}$ (i.e. $t_{\rm cool}(R,t)$) drops to zero is

$$\tau(t) = \frac{\beta_{\rm cool}(t)}{|\dot{\beta}_{\rm cool}|} = \frac{x}{\beta_{\rm cool}(0)}\left(\frac{R}{R_{out}}\right)^{-1.5} t_{\rm cool}(R,t). \quad (4)$$

We adopt three values of x: $x = 10.5$ (fast), $x = 105$ (slow) and $x = 314$ (very slow). In the fast case, $\tau(R,t) \sim t_{\rm cool}(R,t)$ in the outer disc so $t_{\rm cool}$ is changing faster than the disc can come into thermal equilibrium at that value of $t_{\rm cool}$. In the slow case, $\tau(r,t) > t_{\rm cool}(r,t)$ so that the disc is everywhere able to come into thermal equilibrium at that $t_{\rm cool}$. This situation is even more amply satisfied in the very slow case.

For each value of x, we run the simulation until a fragment forms (at $\beta_{\rm cool} = \beta_{\rm frag}$). In those cases where the timing of fragmentation suggests that the slow reduction of $\beta_{\rm cool}$ is acting so as to stabilise the disc at lower $\beta_{\rm cool}$, we test this hypothesis by turning off

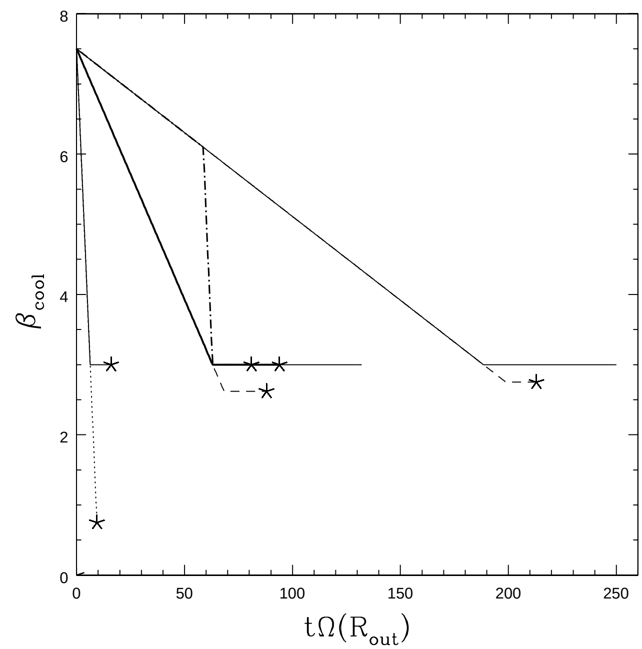

Figure 1. Time evolution of the parameter $\beta_{\rm cool}$ in the various models. The three solid lines refer to cases where $\beta_{\rm cool}$ is first decreased and then held at $\beta_{\rm cool} = 3$ (that is, simulations F2, S1 and VS1). The two dashed lines correspond to simulations S3 and VS2. The dotted line is simulation F1 and the thick solid line is the higher resolution run (Sh). Finally, the dot-dashed line is the simulation with variable rate of change of $\beta_{\rm cool}$ (V). An asterisk at the end of the line indicates fragmentation at this time, whereas runs without an asterisk are unfragmented at the end of the simulation.

the reduction in $\beta_{\rm cool}$ when it attains a value equal to $\beta_{\rm hold}(> \beta_{\rm frag})$. We then experiment with values of $\beta_{\rm hold}$ in order to find the minimum value of $\beta_{\rm hold}$ at which the disc does not fragment over the duration of the numerical experiment. Table 1 and Fig. 1 summarize the main details of the various runs we have performed, where the 'F' simulations are the 'fast' ones, the 'S' are the 'slow' ones and the 'VS' are the 'very slow' ones (see discussion in Section 3 below). The simulation named V was performed with an initially slow reduction of β (with $x = 105$), followed by a fast reduction (with $x = 10.5$), so that it would reach $\beta = 3$ with a fast reduction at the same time as simulation S1. This was run as a control run to ensure that secular evolution did not affect our results (see below).

2.4 Resolution issues

One important aspect that needs to be taken into account is whether the resolution of our simulation is high enough to reproduce fragmentation, when it occurs. Resolution criteria for fragmentation with SPH codes have been discussed by Bate & Burkert (1997). They obtained that SPH correctly reproduces fragmentation if the relevant Jeans mass contains at least 100 SPH particles, that is twice the typical number of neighbours ($N_{\rm neigh} = 50$) within one smoothing region. More recently, Nelson (2006) has revisited this issue focussing on fragmentation in self-gravitating discs and has found a slightly more stringent criterion, requiring that the Jeans mass is resolved with three times as many particles as required by Bate & Burkert (1997). In a gravitationally unstable disc, the most unstable wavelength is given by $\lambda = 2c_{\rm s}^2/G\Sigma$. The Jeans mass (or, as Nelson 2006 calls it, the "Toomre mass") is then given by:

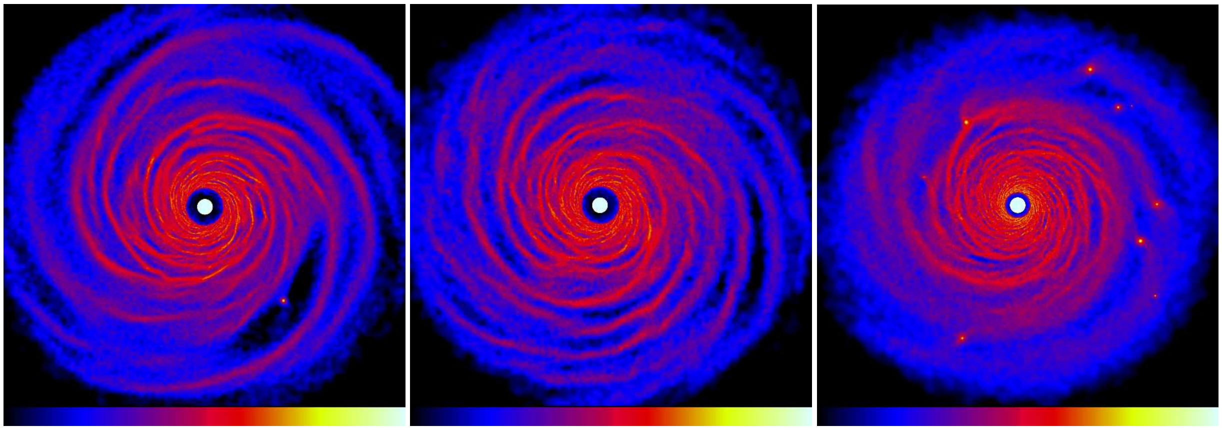

Figure 2. Images of Sh at fragmentation (left) and S1 (centre) at the same time. Although Sh has fragmented, only one fragment is seen at large radius and aside from the immediate area around the fragment the discs are very similar. This should be contrasted with the profusion of fragments in F2 at the point of fragmentation

$$M_J = \pi\Sigma\lambda^2 = \frac{4\pi c_s^4}{G^2\Sigma} = 4\pi^3 Q^2 \left(\frac{H}{R}\right)^2 \Sigma R^2. \quad (5)$$

The cumulative disc mass at radius R is given by:

$$M_{\rm disc}(R) = 2\pi\Sigma R^2 = M_{\rm disc}\frac{R}{R_{\rm out}}, \quad (6)$$

since in our setup $\Sigma \propto R^{-1}$ (see above). We can then rewrite the Jeans mass using eq. (6), as:

$$M_J = 2\pi^2 Q^2 \left(\frac{H}{R}\right)^2 \left(\frac{R}{R_{\rm out}}\right) m_{\rm p} N_{\rm tot}, \quad (7)$$

where we have also used $M_{\rm disc} = m_{\rm p}N_{\rm tot}$, where $N_{\rm tot}$ is the total number of particles used and m_p is the mass of an individual SPH particle. In order to properly resolve fragmentation, we require that $M_T > m_p N_{\rm reso}$, where $N_{\rm reso} = 2N_{\rm neigh} = 100$, according to Bate & Burkert (1997), or $N_{\rm reso} = 6N_{\rm neigh} = 300$ according to Nelson's more restrictive criterion, and recalling that in our simulations the mean number of neighbours per particle is 50. We then obtain that we have enough resolution at radii R that satisfy:

$$\frac{R}{R_{\rm out}} \gtrsim \left(\frac{2}{\pi^2}\frac{1}{Q^4 q^2}\frac{N_{\rm reso}}{N_{\rm tot}}\right)^{1/3}, \quad (8)$$

where $q = M_{\rm disc}/M_\star$ and where we have aso used the relationship between disc thickness and the parameter Q:

$$\frac{H}{R} = \frac{Q}{2}\frac{M_{\rm disc}(R)}{M_\star}, \quad (9)$$

that can be easilty derived from Eq. (1). Based on equation (8) we can then conclude that for $q = 0.1$, as used in the present paper, fragmentation is well resolved at radii $R \gtrsim 5$. Note that, since $N_{\rm reso}$ only enters eq. (8) to the power of one third, if we had used the more restrictive condition of Nelson (2006), we would only increase our minimum radius by a factor 1.4. As shown in Fig. 2, whenever we observed fragmentation, this occurred outside $R \approx 5$, so that we can be confident that we do resolve the relevant mass and length scales for fragmentation.

A second aspect related to resolution is that we require artificial viscosity to play a role only when modeling shocks. In order to ensure this, we then require that the velocity difference accross a smoothing kernel is subsonic, i.e. $h\Omega < c_{\rm s}$, where h is the smoothing length. This in turn requires that the smoothing length is smaller than the disc thickness $H = c_{\rm s}/\Omega$. We have indeed checked that, even at the lower resolution of 250,000 particles, the average smoothing length is a fraction ≈ 0.5 of the disc thickness.

3 RESULTS

We find that in the fast case ($x = 10.5$), a fragment forms when $\beta_{\rm cool} = 0.75$, i.e. about 8.9 outer disc dynamical times after the rapid reduction in $\beta_{\rm cool}$ commenced. Since fragmentation always takes about a dynamical timescale to get under way, it follows that, as expected, the 'fast' case behaves like the usual case where a fixed $\beta_{\rm cool}$ is imposed.

We however see different behaviour in the slow case: here we find that when $\beta_{\rm cool}$ is reduced to, and then held at, $\beta_{\rm hold} = 2.75$ (the evolution of this simulation is not shown in Fig. 1), the disc does not fragment even when the disc is then integrated for a further 44 outer dynamical timescales. Likewise, for the slow case, the disc does not fragment when held at $\beta_{\rm hold} = 3$, even after integration for 63 outer dynamical timescales at this $\beta_{\rm cool}$ value. [1] On the other hand, when $\beta_{\rm cool}$ was instead held at 2.62, it fragmented after a further $\sim$ 18 outer dynamical timescales, so it would appear that the fragmentation boundary is at around 2.7. This is in strong contrast with the value of $\sim$ 7 derived in previous work where a fixed $\beta_{\rm cool}$ is imposed. We hesitate to say that we have proved that a disc will *never* fragment when brought to such a low value of $\beta_{\rm cool}$ value at this slow rate, since our experience shows that where one is close to the limit of marginally stable $\beta_{\rm cool}$, fragmentation may ensue after long timescales, and that its timing may depend on numerical noise that can be affected by resolution. Indeed, we found that when we re-ran the $x = 105$ simulation at higher resolution ($N = 500,000$), and held it at $\beta_{\rm hold} = 3$, it eventually did form a fragment at large radius . We however show the disc structure in this simulation at the point of fragmentation and contrast it with the corresponding situation when the cooling time is rapidly

[1] We have tested whether this resistance to fragmentation in the slow case is simply because the disc takes longer to reach $\beta_{\rm cool} = 3$ and is therefore of lower mass, due to accretion onto the central star. However, in the control run V (in which the disc attains $\beta_{\rm cool} = 3$ at the same time, but with rapid ($x = 10.5$) reduction in $\beta_{\rm cool}$ between $\beta_{\rm cool} = 6$ and $\beta_{\rm cool} = 3$), the disc fragments promptly. Thus we are satisfied that it is indeed the value of x which controls fragmentation.

reduced and then held at constant $\beta_{\rm cool} = 3$ (i.e. model F2). Evidently, notwithstanding the fact that a fragment does eventually form in the former case also, the disc structure is quite different in the two cases, with the 'rapid' simulation containing a number of regions that are on the point of fragmentation at the moment that the first fragment appears. Our interest here is not in defining precise boundaries at which fragmentation will or will not occur (since the definition of such a boundary is always contingent on the duration of the simulation) but in demonstrating that the structure of the disc is indeed affected not just by the instantaneous value of $\beta_{\rm cool}$ (and hence on the heating rate that has to be delivered through the action of the self-gravitating modes) but also on the *history* of how the disc arrived at such a value of $\beta_{\rm cool}$.

This then raises the possibility (which we discussed in Section 1) that the lower limit on $\beta_{\rm cool}$ for self-regulation might represent the difficulties that a disc might have in achieving a self-regulated state on an appropriately short timescale, rather than a fundamental upper limit on the dissipation rate that can be provided by gravitational modes in the absence of fragmentation. In principle, then, we could envisage a situation where the disc might be self-regulated at an arbitrarily low value of $\beta_{\rm cool}$ (i.e. where the non-linear development of the spiral modes delivered an arbitrarily high heating rate without the disc fragmenting) provided that the disc approached this state sufficiently slowly. Such a conclusion would contradict that of Lodato and Rice 2005, who interpreted the fragmentation boundary in terms of a (history independent) limit on the maximum α value delivered by such instabilities.

In order to explore this further, we ran the very slow ($x = 314$) simulations so that we could test whether the disc could remain self-regulated at a yet smaller value of $\beta_{\rm cool}$ than for the $x = 105$ case. We however found little difference in the results for the $x = 105$ and $x = 314$ case, the lowest values at which $\beta_{\rm cool}$ could be held being respectively 2.75 and 3.00 for the two cases (for $N = 250,000$ in both cases).

4 CONCLUSIONS

We have found that the *rate* at which the cooling timescale is changed indeed affects the minimum value of $\beta_{\rm cool}$ at which the disc can exist in a stable, self-regulated state. As expected, this effect is only manifest when the the cooling timescale is varied on a timescale (τ) that is longer than the cooling timescale, since for $\tau < t_{\rm cool}$, the temperature always falls on a timescale $t_{\rm cool}$, irrespective of τ. We find that when $\tau > t_{\rm cool}$, the self-regulated state is sustainable at cooling times that are about a factor two less than those that are possible when a fixed cooling timescale is imposed at the outset of the simulation. This implies that (in the slow cooling case) the gravitational instabilities are able to deliver about twice the heating rate without the disc fragmenting. In terms of the 'viscous alpha' description of such instabilities (Shakura and Sunyaev 1973, Gammie 2001, Lodato & Rice 2005), the maximum α deliverable by such a disc is then increased from ~ 0.06 to ~ 0.12. It should be noted that such 'local' description of the transport induced by gravitational instabilities is only possible in the limit in which global, wave-like transport does not play an important role. Lodato & Rice (2004, 2005), using a cooling prescription similar to ours, have shown that this is the case, as long as the total disc mass is small ($\lesssim 0.2 M_\star$), which is the case for our simulations. Mejia et al. (2005), using a constant cooling time, argue that global effects might be present, but do not explicitly calculate such global torques. On the other hand, recent calculation by Boley et al. (2006) (see in particular their Fig. 13), which employ more realistic cooling properties, confirm that in the limit of small disc mass, the transport induced by gravitational instabilities is essentially local.

We have thus found that thermal history can affect the ability of the disc to exist in a self-regulated state without fragmentation but that this affects the location of the stability boundary at only the factor two level. The fact that there was negligible change in the fragmentation boundary when even slower changes in $t_{\rm cool}$ were employed, demonstrates that thermal history is only part of the story. Our results suggest that, however slowly the disc is cooled through a sequence of thermal equilibria, there is still a fundamental upper limit to the heating that can be provided by gravitational instabilities in a non-fragmenting disc. Thus it would appear that the initiation of fragmentation in a self-gravitating disc does *not* require that the disc enter the regime of rapid cooling on a *short* timescale. It is thus unnecessary to invoke sudden events (e.g. impulsive interactions with passing stars, Lodato et al. 2007) to tip a previously self-regulated disc into the fragmenting regime. Instead our results suggest that fragmentation can in principle be approached via the *secular* evolution of a self-gravitating disc.

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